A. In Refereed Journals

2023 Charalambous N., Lu Z. *'Connected essential spectrum: The case of differential forms.'*
arXiv:2106.01992, To appear, Israel Journal of Mathematics

Abstract. In this article we prove that, over complete manifolds of dimension n with vanishing curvature at infinity, the essential spectrum of the Hodge Laplacian on differential k-forms is either empty, or a connected interval for $0 \leq k \leq n$. The main idea behind the proof is to show that large balls of these manifolds, which capture their spectrum, are close in the Gromov-Hausdorff sense to product manifolds. We achieve this by carefully describing the collapsed limits of these balls. Then, via a new generalized version of the classical Weyl criterion, we demonstrate that very rough test forms that we get from the ε-approximation maps can be used to show that the essential spectrum is a connected interval. We also prove that, under a weaker condition where the Ricci curvature is asymptotically nonnegative, the essential spectrum on k-forms is $[0, \infty)$, but only for $0 \leq k \leq q$ and $n-q \leq k \leq n$ for some integer $q \geq 1$ which depends the structure of the manifolds at infinity. Our results can also be generalized to Schrödinger operators with double-well potential.

2023 Charalambous N., Große N. *'A note on the spectrum of magnetic Dirac operators.'*
arXiv:2306.00590, Submitted

Abstract. In this article we study the spectrum of the magnetic Dirac operator, and the magnetic Dirac operator with potential over complete Riemannian manifolds. We find sufficient conditions on the potentials as well as the manifold so that the spectrum is either maximal, or discrete. We also show that magnetic Dirac operators can have a dense set of eigenvalues.

2023 Charalambous N., Rowlett J. *'The Laplace spectrum on conformally compact manifolds.'*
arXiv:2306.09291, Submitted

Abstract. We consider the spectrum of the Laplace operator acting on L^p over a conformally compact manifold for $1 \leq p \leq \infty$. We prove that for $p \neq 2$ this spectrum always contains an open region of the complex plane. We further show that the spectrum is contained within a certain parabolic region of the complex plane. These regions depend on the value of p, the dimension of the manifold, and the values of the sectional curvatures approaching the boundary.

2023 Charalambous N., Grosse N. *'On the L^p spectrum of the Dirac Operator.'*
Journal of Geometric Analysis 44: 24pp, 2023

Abstract. Our main goal in this paper is to expand the known class of noncompact manifolds over which the L^2-spectrum of a general Dirac operator and its square is maximal. To achieve this, we first find sufficient conditions on the manifold so that the L^p-spectrum of the Dirac operator and its square is independent of p for $p \geq 1$. Using the L^1-spectrum, which is simpler to compute, we generalize the class of manifolds over which the L^p-spectrum of the Dirac operator is the real line for all p. We also show that by applying the generalized Weyl criterion, we can find large classes of manifolds with asymptotically nonnegative Ricci curvature, or which are asymptotically flat, such that the L^2-spectrum of a general Dirac operator and its square is maximal.

2022 Charalambous N., Lu Z. *'A note on Cheeger's isoperimetric constant.'*
Journal of Geometric Analysis 32: 14pp, 2022

Abstract. In this article we provide a simplified proof of Buser's result for Cheeger's isoperimetric constant, namely that

$$\lambda_1(M) \leq C(n) \left(\sqrt{K}\, h(M) + h^2(M)\right),$$

whenever M is a smooth compact manifold with Ricci curvature bounded below, $\mathrm{Ric} \geq -(n-1)K$. Here $\lambda_1(M)$ is the first nonzero eigenvalue of the Laplacian on functions, $h(M)$ is Cheeger's constant, and $C(n)$ is a constant that depends only on the dimension of the manifold. We achieve this by proving a more flexible comparison result about the area of a hypersurface within a larger geodesic ball, in comparison to its volume within the ball. This comparison allows us to avoid having to resort to the consideration of Dirichlet regions. We also provide a comprehensive approach on how to obtain volume estimates for smooth hypersurfaces.

2020 Charalambous N., Lu, Z. *'The spectrum of the Laplacian on forms over flat manifolds.'*
Mathematische Zeitschrift 296:1-12, 2020

Abstract. In this article we prove that the spectrum of the Laplacian on k-forms over a noncompact manifold flat is always a connected closed interval of the nonnegative real line. The proof is based on a detailed decomposition of the structure of flat manifolds at infinity. It allows us to prove that even on flat manifolds the k-form spectrum can have a positive lower bound. The bottom of the spectrum reflects more details about the structure of the manifold at infinity which are missed by the function spectrum.

2020 Charalambous N. *'The Yang-Mills heat equation on three-manifolds with boundary.'*
J. Stoch. Anal. 1, no. 4, Art. 10, 23 pp., 2020

Abstract. In this invited survey article we provide an expository account of the work of Leonard Gross and the author on the Yang-Mills heat equation over smooth three-manifolds with boundary.

2020 Charalambous N., Leal, H., Lu, Z. *'Spectral Gaps on Complete Riemannian manifolds.'*

Contemporary Mathematics 756: 57–67, 2020.

Abstract. In this article, after surveying some basic results related to the generalized Weyl criterion for the essential spectrum, we use the language of Gromov-Hausdorff convergence to prove a spectral gap theorem. In particular, by appropriately gluing together compact manifolds and taking the limit in the Gromov-Hausdorff sense, we construct a smooth periodic manifold which will has gaps in its essential spectrum. The proof makes use of our generalized Weyl criterion.

2019 Charalambous N., Lu, Z. *'The spectrum of continuously perturbed operators and the Laplacian on forms.'*

Differential Geometry and Applications 65:227–240, 2019

Abstract. In this article we study the variation in the spectrum of a self-adjoint nonnegative operator on a Hilbert space under continuous perturbations of the operator. In the particular case of the Laplacian on k-forms over a complete manifold we use this analytic result to obtain some interesting and significant properties of its spectrum. In particular, we prove the continuous deformation of the spectrum of the Laplacian under a continuous deformation of the metric of the noncompact manifold. We also show that the spectrum on 1-forms always contains the function spectrum on any open manifold. Both results are analytic ones, which do not impose any assumptions on the curvature nor the volume growth of the manifold. The second one also implies that we do not always have to make stronger geometric assumptions on the manifold to compute the k-form spectrum, in comparison to the Laplacian on functions.

2019 Charalambous N., Rowlett J. *'The heat trace for the drifting Laplacian and Schrödinger operators on manifolds.'*

Asian Journal of Mathematics 23: 539–559, 2019

Abstract. We study the heat trace for Schrödinger operators and the drifting Laplacian on compact Riemannian manifolds. When the potential (resp. weight function) is bounded and measurable we prove the existence of a partial asymptotic expansion of the heat trace for small times and a suitable remainder estimate. This expansion is sharp in the sense that further terms in the expansion exist if and only if the potential is of higher Sobolev regularity. In the case of a smooth weight function, we determine the full short-time asymptotic expansion of the heat trace for the drifting Laplacian. We use this expansion to show that the Weyl law coincides with the Weyl law for the standard Laplace-Beltrami operator. We also prove isospectrality results for the drifting Laplacian.

2017 Charalambous N., Gross, L. *'Initial Behavior of Solutions to the Yang-Mills Heat Equation.'*
Journal of Mathematical Analysis and Applications 451:873–905, 2017.

Abstract. Long time existence and uniqueness of solutions to the Yang-Mills heat equation have been proven over a compact 3-manifold with boundary for initial data of finite energy. In the present paper we improve on previous estimates by using a Neumann domination technique that allows us to get much better pointwise bounds on the magnetic field. As in the earlier work, we focus on Dirichlet, Neumann and Marini boundary conditions. In addition, we show that the Wilson Loop functions, gauge invariantly regularized, converge as the parabolic time goes to infinity.

2015 Charalambous N., Lu Z. *'Heat kernel estimates and the essential spectrum on weighted manifolds.'*
Journal of Geometric Analysis 25:536-563, 2015

Abstract. We consider a complete noncompact smooth Riemannian manifold M^n with a weighted measure (smooth metric measure space) and the associated drifting Laplacian. We demonstrate that whenever the q-Bakry-Émery Ricci tensor on M is bounded below, then we can obtain an upper bound estimate for the heat kernel of the drifting Laplacian from the upper bound estimates of the heat kernels of the Laplacians on a family of related warped product spaces. We apply these results to study the essential spectrum of the drifting Laplacian on M.

2015 Charalambous N., Gross L. *'Neumann Domination for the Yang-Mills Heat Equation.'*
Journal of Mathematical Physics 56, 073505, 2015

Abstract. Long time existence and uniqueness of solutions to the Yang-Mills heat equation have been proven over a compact 3-manifold with boundary for initial data of finite energy. In the present paper we improve on previous estimates by using a Neumann domination technique that allows us to get much better pointwise bounds on the magnetic field. As in the earlier work, we focus on Dirichlet, Neumann and Marini boundary conditions. In addition, we show that the Wilson Loop functions, gauge invariantly regularized, converge as the parabolic time goes to infinity.

2015 Charalambous N., Lu Z. *'The L^1 Liouville Property on Weighted Manifolds.'*
Contemporary Mathematics 653:65-79, 2015

Abstract. We consider a complete noncompact smooth metric measure space $(M^n, g, e^{-f}dv)$ and the associated drifting Laplacian. We find sufficient conditions on the geometry of the space so that every nonnegative f-subharmonic function with bounded weighted L^1 norm is constant.

2014 Charalambous N., Lu Z. *'On the Spectrum of the Laplacian.'*
Mathematische Annalen 359:211-238, 2014

Abstract. In this article we prove a generalization of Weyl's criterion for the essential spectrum of a self-adjoint operator on a Hilbert space. The proof uses the spectral decomposition of the operator in order to rewrite our assumptions. We then apply this criterion to the Laplacian on functions over open manifolds and get new results for its essential spectrum. In particular, we show that the L^2 essential spectrum of the Laplacian over a complete shrinking Ricci soliton is $[0, \infty)$ without any further assumptions on its curvature. We also prove that the L^2 essential spectrum is $[0, \infty)$ on a complete manifold whose radial Ricci curvature is asymptotically nonnegative and whose volume does not decay exponentially at a point.

2013 Charalambous N., Gross L. *'The Yang-Mills Heat Semigroup on Three-Manifolds with Boundary.'*
Communications in Mathematical Physics 317:727-785, 2013

Abstract. Long time existence and uniqueness of solutions to the Yang-Mills heat equation is proven over a compact 3-manifold with smooth boundary. The initial data is taken to be a Lie algebra valued connection form in the Sobolev space H_1. Three kinds of boundary conditions are explored, Dirichlet type, Neumann type and Marini boundary conditions. The last is a nonlinear boundary condition, specified by setting the normal component of the curvature to zero on the boundary. The Yang-Mills heat equation is a weakly parabolic non-linear equation. We use gauge symmetry breaking to convert it to a parabolic equation and then gauge transform the solution of the parabolic equation back to a solution of the original equation. Apriori estimates are developed by first establishing a gauge invariant version of the Gaffney-Friedrichs inequality. A gauge invariant regularization procedure for solutions is also established. Uniqueness holds upon imposition of boundary conditions on only two of the three components of the connection form because of weak parabolicity. This work is motivated by possible applications to quantum field theory.

2010 Charalambous N. *'Eigenvalue Estimates for the Bochner Laplacian and Harmonic Forms on Complete Manifolds.'*
Indiana University Mathematics Journal 59:183-206, 2010

Abstract. We study the set of eigenvalues of the Bochner Laplacian over a geodesic ball of an open manifold M, and find lower estimates for these eigenvalues when M satisfies a Sobolev inequality. We show that we can use these estimates to demonstrate that the set of harmonic forms of polynomial growth over M is finite dimensional, under sufficient curvature conditions. We also study in greater detail the dimension of the space of bounded harmonic forms on coverings of compact manifolds.

2007 Charalambous N. *'On the Equivalence of Heat Kernel Estimates and Logarithmic Sobolev Inequalities for the Hodge Laplacian.'*
Journal of Differential Equations 233:291-312, 2007

Abstract. In this paper we consider the Laplacian on differential k-forms over a smooth open manifold M^n, not necessarily compact. We find sufficient conditions under which the existence of a family of logarithmic Sobolev inequalities for the Laplacian on k-forms is equivalent to the ultracontractivity of its heat operator. We will also show how to obtain a logarithmic Sobolev inequality for the Laplacian on k-forms when there exists one for the Laplacian on functions. In the particular case that the Ricci curvature is bounded below, we use the Gaussian type bound for the heat kernel of the Laplacian on functions in order to obtain a similar Gaussian type bound for the heat kernel of the Laplacian on k-forms. This is done via logarithmic Sobolev inequalities and under the additional assumption that the Weitzenböck tensor on k-forms and volume of balls of radius one are uniformly bounded below.

2007 Charalambous N. *'L^2 Harmonic Forms and the Structure of Complete Manifolds.'*
Proceedings of the International Conference in Memory of José F. Escobar,
Revista Matemáticas: Enseñanza Universitaria de la ERM, 2007.

Abstract. In this paper we discuss the relation between the existence of L^2 harmonic one-forms on a complete noncompact manifold and its structure. In particular, we prove an improved Bochner inequality for L^2 harmonic one-forms and demonstrate that in the equality case the set of L^2 harmonic one-forms spans a totally geodesic submanifold.

2005 Charalambous N. *'On the L^p Independence of the Spectrum of the Hodge Laplacian on Non-Compact Manifolds.'*
Journal of Functional Analysis 224:22-48, 2005

Abstract. The central aim of this paper is the study of the spectrum of the Laplacian on differential k-forms in L^p. This is done over a smooth open manifold M^n with Ricci Curvature bounded below and uniformly subexponential volume growth. It will be demonstrated that on such manifolds the L^p spectrum of the Laplacian on k-forms is independent of p for $1 \leq p \leq \infty$, whenever the Weitzenböck Tensor on k-forms is also bounded below. It follows as a corollary that the isolated eigenvalues of finite multiplicity are L^p independent. The proof relies on the existence of a Gaussian upper bound for the heat kernel of the Laplacian on k-forms. By considering the L^p spectra on the Hyperbolic space H^{N+1} we conclude that the subexponential volume growth condition is necessary in the case of 1-forms. As an application, we will show that the spectrum of the Laplacian on 1-forms has no gaps on certain manifolds with a pole or that are in a warped product form. This will be done under less strict curvature restrictions than what has been known so far and it was achieved by computing the L^1 spectrum of the Laplacian and applying the L^p independence result.

B. Book Chapters

2015 Charalambous N., Lu Z., Rowlett J. *'Eigenvalue Estimates on Bakry-Émery Manifolds.'*
Elliptic and Parabolic Equations, Springer Proceedings in Mathematics and Statistics 119:45-61, 2015

Abstract. We demonstrate lower bounds for the eigenvalues of the drifting Laplacian over compact Bakry-Émery manifolds (compact smooth metric measure spaces) with and without boundary. The lower bounds for the first eigenvalue rely on a generalised maximum principle which allows gradient estimates in the Riemannian setting to be directly applied to the Bakry-Émery setting. Lower bounds for all eigenvalues are demonstrated using heat kernel estimates and a suitable Sobolev inequality.

C. Preprints

'On a Weyl Criterion for the L^p spectrum.' with Z. Lu.

D. Work in Progress

'Scattering Theory for operators on differential forms over asymptotically Euclidean manifolds.' with C. Aldana, A. Strohmaier, et. al.

'The heat kernel and spectrum of symmetric spaces.' with P. Siasos.